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We attach an annual report describing progress of work which we have done on five projects. Four papers which are enclosed with the report have been submitted for publication and they should serve as expanding and giving more background for the description of the work in the report. The report also details all activities, such as conference attendence, visits, visitors, etc., which have been undertaken with the support of the grant.

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Convergence and Performance of Synchronous and Asynchronous Parallel and Conventional Iterative Methods

ANNUAL REPORT

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THE ANNUAL REPORT:

RESEARCH PROGRESS AND ACTIVITIES

This report is divided into three parts. *In the first part* we describe briefly five research projects that we have pursued during the course of 1988. Where our work has resulted in a paper which was submitted for publication, the paper is attached at the end of this report and should be regarded as providing expansion and background for the description of work given below. Further background material can be found in the semi–annual report submitted to the AFOSR on July 6, 1988 and also in the new proposal submitted to AFOSR on September 2, 1988. *In the second part* we detail the activity that has been undertaken with the full or partial support of the grant. This includes conferences attended, talks given, visits to consult specialists in other universities and specialists from other institutions who have come to the University of Connecticut to give consultancy to the P. I. in connection with the research projects. *In the third part* we list the papers which were submitted for publication and which grew out of our work supported by the grant. They are arranged as a "reference" list to which we refer from the earlier parts of this report.

PART 1: Research Progress

In the calendar year 1988 our work which was supported by Grant AFOSR-88-0047 concentrated on the following topics:

i) We have observed that when applying *parallel iterative methods* to a *singular* but solvable system of equations $Ax = b$, such methods may converge to limit points which fail to be solutions to the system. A situation in which an iterative method has limit points which are not solutions to a linear system which is desired to be solved is called in the literature *inconsistency of the iterative method with the linear system.* In theory, when there is no round-off error is present, inconsistency does not arise when *sequential* iterative methods are applied to determine solutions to a solvable system. In the paper referenced [3] below, which was submitted and accepted for

publication, we investigate the reason for the loss of consistency caused by the introduction of the parallel multisplitting method. Then, for the class of singular $n \times n$ M-matrices A which frequently arise in applications, we characterize by a matrix combinatorial approach multisplittings which induce parallel iterative methods guaranteed to be consistent.

The work on this project was carried out with Professor J. Philip Kavanagh who was visiting the University of Connecticut during the Academic Year 1987/88. Half of his salary was paid by money from the P. I. grant as arranged between AFOSR, the University of Connecticut, and the P. I.

ii) The Kaczmarz algorithm is a method for solving $m \times n$ linear systems $Ax = b$ which views the solutions, when they exit, as lying in the intersection of m hyperplanes. A solution is obtained from an initial vector by cyclically projecting the current approximation orthogonally onto the next hyperplane. Variations of the Kaczmarz algorithm such as the ART–SOR (algebraic reconstruction successive overrelaxtaion) method are currently used by computer tomographists in image reconstruction from incomplete data, that is, when images of the objects are available along relatively few directions (for example in well-to-well tomography). We have investigated and proved the convergence of an asynchronous parallel implementation of the ART–SOR method. The proof of convergence involves the embedding of the process in a higher dimensional space and then applying some delicate arguments using dual norms. Professor Ludwig Elsner of the University of Bielefeld in West Germany came for a few days' visit to the University of Connecticut and, together with Professor I. Koltracht of our department, we concentrated on producing a preliminary report on this work. A paper summarizing our results, see [1], has now been submitted to the special issue of Linear Algebra and its Applications on Image Reconstruction from Projections. We are in the preliminary stages of finding out whether we can use the SIMD machine (a Connection 2) and a MIMD simulator at the Computer Center of United Technologies to experiment with this method.

iii) We have made good progress in our investigation of *when* initial points of trajectories of systems of differential equations $\dot{x} = Ax$ which eventually become and remain nonnegative *can be discovered* by finite differences approximations whose step-size does not have to become increasingly small. *At first we showed that* when $A = (a_{i,j})$ is an essentially nonnegative matrix (i.e. $a_{i,j} \geq 0 \; \forall i \neq j$) and A has either a real spectrum or its generalized eigenspace of corresponding to the spectral abscissa contains a positive vec-

tor, *then* there exists an $\hat{h}(A)$ such that for any time-step $0 < h < \hat{h}(A)$ all initial points for such trajectories can be discovered by finite differences approximations with time-step h. These results were summarized in paper [4] which was submitted and accepted for publication. Since then together with one of the graduate students supported by the grant, Mr. Michael Tsatsomeros, we have proved that every essentially nonnegative matrix (regardless of any additional spectral properties) possesses such an $\hat{h}(A)$. We are currently continuing with the investigation and the implications of this property.

iv) Together with Martin Hanke of the University of Karlsruhe in West Germany who came to visit the University of Connecticut for five days in November 1988 we continued our investigation of when iterative methods which are derived from a *preconditioning* of an $m \times n$ linear system $Ax = b$ are guaranteed to converge to an approximate solution to the system. We began this work at a NATO Workshop on Numerical Linear Algebra, Digital Signal Processing, and Parallel Algorithms which was held in Belgium in August, 1988. We think that the solution of this problem will permit us to develop an algebraic approach to the question of consistency of parallel iterative algorithm for solving singular systems which is an alternative to the combinatorial approach developed in (i).

v) The project leading to [3] involved much work with the combinatorial properties of nonnegative matrix A which relate to its generalized eigenspace corresponding to its spectral radius (that is, corresponding to its Perron eigenvalue). We have examined the possibility of generating a basis for this eigenspace which consists entirely of nonnegative vectors by means of asymptotic expansions. The work began with Professors R. E. Hartwig and N. J. Rose of North Carolina State University. Professor Hartwig came to visit the University of Connecticut for a few days in June, 1988, and we were able then to refine this process to the extent that we now have a concrete algorithm. (See Section 2 of [2], in particular Theorem 2.2.) A graduate assistant, Ms. Emei Wang, is now investigating the stability of this algorithm.

PART 2: Activities

During 1988 we have discussed or presented the results of AFOSR-88-0047 at the following meetings: (a) Special Session on "Modern Trends in

Matrix Analysis and Applications", American Mathematical Society Annual Meeting, Atlanta, 1/6/88–1/9/88; (b) "The Oberwolfach Meeting on Numerical Linear Algebra and Parallel Computations", Oberwolfach, West Germany 2/29/88–3/4/88; (c) Workshop on "Iterative Solutions of Singular Linear Systems of Equations", Institute of Practical Mathematics, University of Karlsruhe, West Germany, 3/7/88; (d) Minisymposium on "Iterative Methods for the Solution to Linear Algebraic Systems 5", Third SIAM Conference on Applied Linear Algebra, Madison, Wisconsin, 5/23/88–5/26/88; (e) "International Congress on Computational and Applied Mathematics", University of Leuven, Belgium, 7/25/88–7/29/88; (f) "NATO Advanced Study Institute on Numerical Linear Algebra, Digital Processing , and Parallel Algorithms", Leuven, Belgium, 8/1/88–8/12/88; (g) "Conference on Iterative Methods for Large Linear Systems", University of Texas at Austin, 10/19/88–10/21/88.

The meeting in Oberwolfach was most useful. There the proposer was able to work intensively with Professor Ludwig Elsner of the University of Bielefeld in West Germany. Consequently, Professor Elsner came to visit Connecticut in the Fall to work on a preliminary report summarizing our results.

In January, 1988, the proposer visited North Carolina State University to consult with Professors Robert J. Plemmons, Robert E. Hartwig and Nicholas J. Rose. During the visit he was able to obtain an algebraic proof for the existence of a nonnegative basis for the Perron eigenspace of a nonnegative matrix. The proposer expects this work to complement the combinatorial approach to the questions of consistency and convergence of the synchronous parallel multisplitting iteration process for solving systems whose coefficient matrix is a singular M–matrix as in (i) above.

PART 3: Reports and Publications

The work stemming from the AFOSR proposal No. 88-0047 has resulted in the submission for publication of the following papers:

[1] (with L. Elsner and I. Koltracht) "On the convergence of asynchronous paracontractions with applications to tomographic reconstruction from incomplete data." Submitted for publication in Lin. Alg. Appl.

[2] (with R. E. Hartwig and N. J. Rose) "An algebraic–analytical approach